

Evaluating Public Bus Shelter Service Quality by Using Complex Neutrosophic Einstein Prioritized Operators

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ABSTRACT

A robust decision-making strategy is essential for efficiently collecting and evaluating complex data, yet in real-world urban settings, ambiguity often prevails. To address these challenges, this paper introduces a multi-criteria group decision-making (MCGDM) algorithm specifically designed for the complex single-valued neutrosophic (CSVN) environment. The CSVN framework offers greater consistency and reliability than intuitionistic fuzzy sets (IFS) by handling a triplet of truthiness, indeterminacy, and falsity, which better captures indeterminate and inconsistent human judgments. Recognizing the critical role of aggregation operators in synthesizing diverse criteria into a single evaluative value, we develop novel Einstein Prioritized Aggregation Operators (EPAO) for CSVN elements. Specifically, the complex single-valued neutrosophic Einstein prioritized weighted averaging (CSVNEPWA) and weighted geometric (CSVNEPWG) operators are presented, and their distinctive properties are examined. Leveraging these operators, a comprehensive MCGDM algorithm is constructed to navigate uncertain decision environments. As a real-world illustration, the model is applied to evaluate the quality of the public Metrobus system's shelter services in Istanbul, Turkey. Five distinct shelter alternatives are assessed against five strategic criteria: weather protection, seating comfort, cleanliness and maintenance, information systems, and safety and lighting by a panel of urban mobility experts. The results confirm the model's capacity to prioritize infrastructure investments under deep ambiguity, offering a transparent, adaptive tool for transit authorities to enhance passenger experience.

1. Introduction

The quality of bus shelter infrastructure profoundly shapes user experience, perceived safety, and overall satisfaction with public transport systems. In high-capacity bus rapid transit (BRT) networks

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like Istanbul's Metrobus, shelters serve not merely as waiting areas but as critical nodes that mediate passenger comfort, information access, and modal attractiveness. Despite extensive research on service quality in public transportation [24-26], shelter-specific attributes, such as weather protection, seating, cleanliness, real-time information, and lighting, are often subsumed under generic station environment variables, limiting targeted improvement strategies. Cervero [8] argued that high-quality waiting facilities directly enhance BRT ridership by reducing the psychological costs of transfer and waiting time. Eboli and Mazzulla [11] demonstrated that structural comfort and information provision significantly influence customer satisfaction indices, while Hensher et al. [14] stressed that perceived shelter quality affects willingness to pay for premium transit services. For Istanbul's Metrobus, which carries over a million passengers daily, systematically evaluating shelter service quality is essential to guide investment prioritization and align infrastructure with rising user expectations. This study develops a multi-criteria evaluation framework specifically tailored to Metrobus shelters, providing actionable insights for IETT planners and decision-makers.

Effectively capturing these qualitative service attributes requires a systematic way to manage the inherent imprecision and subjective judgment of decision-makers, a challenge that has long motivated the search for formal representations of uncertainty. Scholars have studied the connection between precision and ambiguity for a long period. In response to this challenge, Zadeh introduced the concept of fuzzy sets (FS) [34] to address ambiguity and imprecision. FS manages imprecise and uncertain data by assigning membership degrees (MDs) to individual elements, which take values in $[0,1]$. However, a limitation of FS functions is that they only operate on MD values and do not incorporate non-membership degree (NMD). Limitations of the FS model led to the creation of the intuitionistic fuzzy set (IFS) by Atanassov (Atanassov, 1986), which represents each member through two numbers from $[0, 1]$ to express both MD and NMD simultaneously. Researchers have taken an interest in this theoretical framework because it enables effective expression of fuzzy decision information. Nonetheless, given IFS's confined scope within the $[0, 1]$ range, researchers are inclined to explore alternative theories that offer greater flexibility. For this, Smarandache [33] presented the notion of the neutrosophic fuzzy set (NS), which consists of a triplet that includes truthiness, indeterminacy, and falsity over $(0^-, 1^+)$. Since it is a non-standard interval, it is difficult to apply it to real-world problems. To overcome this, various classes of NS were defined, including the single-valued neutrosophic set (SVNS) [19], complex neutrosophic set (CNS) [3], hesitant neutrosophic set (HNS) [16], and then these classes were used by the researcher for aggregating decision-making problems [12], digital image processing [20], similarity measures [9], distance measure [30], medical image analysis [22], etc.

To address decision-maker (DM) problems, two techniques were typically discussed: traditional methods, such as the technique for order of preference by similarity to ideal solution (TOPSIS) [31], and MARCOS Methodology [15]. At the same time, the other one is the aggregate operators (AOs), which are more effective for handling imprecise data than traditional approaches. AOs are considered more efficient and reliable because they provide full information on the alternatives, including ranking values, whereas prior techniques can only provide ranking values. This difference drew the attention of researchers and experts; as a result, AOs are now the subject of evaluation in decision-making domains. Several methods have been proposed to aggregate the data, including intuitionistic fuzzy (IF) weighted [2], hybrid averaging operators [1], Dombi AO [4], Muirhead mean AO [17], and geometric AO [21]. Ali et al. [5] define the TOPSIS method by utilizing the Bonferroni mean (BM) AO for the multi-criteria decision-making (MCDM) problem, and Pamucar et al. [28] define the weighted geometric BM operator. Zhang et al. [24] defined the Dombi weighted AO for single-valued neutrosophic elements (SVNE), and Majumder et al. [23] proposed an intuitionistic

fuzzy-based hybrid decision-making approach to determine the priority values of indicators and their application to solar energy feasibility analysis. All these AOs assist the DM by aggregating data and supporting decision-making.

However, MCDM is a problem frequently encountered across disciplines, including the social sciences, economics, management, and medicine. DM is struggling to make sense of the expert's ambiguous information. This is one of the most challenging tasks individuals face when dealing with the complexity of MCDM. In this paper, to handle such kind of ambiguous information, we have introduced the complex single-valued neutrosophic set (CSVNS) for the multi-criteria group decision-making (MCGDM) problem based on the Einstein Prioritized Aggregate Operator (EPAO), which gives a more effective and efficient value by aggregating the information provided by the DM in a CSVN environment and assists them in selecting the best alternative. A real-life investigation of the quality of shelter services in the public Metrobus system has been adopted. The need for this research arises because optimization and manufacturing integration remain problematic, making it difficult to achieve. Traditional CIM systems cause two major issues in manufacturing: requiring specialized expertise and resulting in high implementation costs and complex designs [28]. The systems need to integrate with emerging technologies such as AI and machine learning (ML). The research proposal addresses existing gaps by developing a novel framework for CIM that leverages modern technologies. The developed framework promotes greater manufacturing process efficiency and improved flexibility.

1.1. Motivation and Objectives of the Study

The evaluation of data in everyday decision-making becomes complex due to ambiguous information, which requires effective assessment strategies. Existing approaches, including IFS, do not adequately resolve problems that occur in real-world scenarios. The current requirements demand an advanced algorithm that supports decision-making under uncertainty. Based on the above analysis, this paper develops the CSVNS framework to address MCGDM problems and their ambiguous data-handling requirements. The CSVNS decision system, which utilizes the EPAO, enables more efficient decision-making by leveraging collective input from decision-makers (DMs) within a CSVN framework. This research work has the following primary objectives;

- 1) The formation of EPAO within the CSVN system is used to construct effective and efficient decision-making by aggregating the data in complex settings.
- 2) To present the theory of Einstein prioritized aggregate operator for the CSVN environment.
- 3) Investigate some essential properties and features of the proposed theory and identify the tool for handling decision-making algorithms.
- 4) Demonstrating the flexibility and efficiency of the proposed theory over IFS for handling data.
- 5) Demonstrating the superiority of the proposed approach by comparing it with prior information and validating its effectiveness.

1.2. Contribution of the Study

This paper develops a decision-making algorithm for evaluating the problem within the CSVN framework. The CSVN framework offers greater capabilities than IFS, as it simultaneously handles truthiness, indeterminacy, and falsity.

Our methodology introduces EPAO, which implements operators such as the complex single-valued neutrosophic Einstein prioritized weighted averaging (CSVNEPWA) and the complex single-valued neutrosophic Einstein prioritized weighted geometric (CSVNEPWG), both defined for CSVN elements. The combination of operators produces single values for assessment, enhancing data evaluation in the decision-making process. The proposed AO serves as the foundation to develop an MCGDM algorithm that provides an effective solution method for CSVN environments. Our methodology demonstrates practical effectiveness by applying it to a realistic scenario, demonstrating its capability for real-world implementation. The last part shows how our approach differs from existing methods through an evaluation process that showcases its novel advantages. Moreover, its ability to handle uncertain data makes this method superior to existing approaches, which enables decision-makers to tackle complex daily decision-making situations effectively.

1.3. Layout of the Study

This paper provides the following: Section 2 reviews basic definitions and fundamental operators. Section 3 examines some EPAOs under the CSVN environment and discusses critical theorems and properties. Section 4 defined the MCGDM algorithm for solving the DM problem. Finally, we illustrate a real-world application to assess the validity of the proposed work, and then compare the proposed theory with prior ones in Section 5.

2. Preliminaries

A structured MCDM approach was adopted to evaluate the quality of Istanbul Metrobus shelter services. The methodology integrates expert judgment with quantitative rating techniques to assess alternative shelter configurations against predefined performance criteria. First, five evaluation criteria were identified through a review of transit quality literature and İETT's operational priorities: weather protection, seating comfort, cleanliness/maintenance, passenger information systems, and safety/lighting. Importance weights were assigned by experts in urban transport mobility in Istanbul, reflecting strategic service goals.

In this section, some basic concepts related to CSVNS have been defined. These notions give the essential foundation for the rest of the article.

Definition 1: [6] A CNS $\tilde{z}$ over a universal set $\mathcal{U}$ is characterized as:

$$\tilde{z} = \{ \langle \sigma, (\mathfrak{t}_{\tilde{z}}(\sigma), I_{\tilde{z}}(\sigma), \mathfrak{f}_{\tilde{z}}(\sigma)) | \sigma \in \mathcal{U} \rangle \}, \quad (1)$$

where $\mathfrak{t}_{\tilde{z}}(\sigma) = (\mathfrak{t}_{\tilde{z}_r}(\sigma), \mathfrak{t}_{\tilde{z}_i}(\sigma))$, $I_{\tilde{z}}(\sigma) = (I_{\tilde{z}_r}(\sigma), I_{\tilde{z}_i}(\sigma))$, $\mathfrak{f}_{\tilde{z}}(\sigma) = (\mathfrak{f}_{\tilde{z}_r}(\sigma), \mathfrak{f}_{\tilde{z}_i}(\sigma))$ such that, $\mathfrak{t}_{\tilde{z}}(\sigma), I_{\tilde{z}}(\sigma), \mathfrak{f}_{\tilde{z}}(\sigma) \in (0^-, 1^+)$ and $0^- \leq \mathfrak{t}_{\tilde{z}}(\sigma) + I_{\tilde{z}}(\sigma) + \mathfrak{f}_{\tilde{z}}(\sigma) \leq 3^+$ and $0^- \leq \mathfrak{t}_{\tilde{z}}(\sigma) + I_{\tilde{z}}(\sigma) + \mathfrak{f}_{\tilde{z}}(\sigma) \leq 3^+$.

Moreover, the triplet $(\mathfrak{t}, I, \mathfrak{f})$ defines the membership, indeterminacy, and non-membership of CNS $\tilde{z}$.

Definition 2: [6] A CSVNS $\tilde{z}$ over a set $\mathcal{U}$ is defined as;

$$\tilde{z} : \{ \langle \sigma, (\mathfrak{t}_{\tilde{z}}(\sigma), I_{\tilde{z}}(\sigma), \mathfrak{f}_{\tilde{z}}(\sigma)) | \sigma \in \mathcal{U} \rangle \} \quad (2)$$

where $\mathfrak{t}_{\tilde{z}}(\sigma) = (\mathfrak{t}_{\tilde{z}_r}(\sigma), \mathfrak{t}_{\tilde{z}_i}(\sigma))$, $I_{\tilde{z}}(\sigma) = (I_{\tilde{z}_r}(\sigma), I_{\tilde{z}_i}(\sigma))$, $\mathfrak{f}_{\tilde{z}}(\sigma) = (\mathfrak{f}_{\tilde{z}_r}(\sigma), \mathfrak{f}_{\tilde{z}_i}(\sigma))$ such that, $0^- \leq \mathfrak{t}_{\tilde{z}}(\sigma) + I_{\tilde{z}}(\sigma) + \mathfrak{f}_{\tilde{z}}(\sigma) \leq 3^+$ and $0^- \leq \mathfrak{t}_{\tilde{z}}(\sigma) + I_{\tilde{z}}(\sigma) + \mathfrak{f}_{\tilde{z}}(\sigma) \leq 3^+$.

Definition 3:[6] Let $\tilde{z}_1 = \left(\left(\mathfrak{t}_{\tilde{z}_{r1}}(\sigma), \mathfrak{t}_{\tilde{z}_{i1}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{r1}}(\sigma), \mathfrak{l}_{\tilde{z}_{i1}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{r1}}(\sigma), \mathfrak{f}_{\tilde{z}_{i1}}(\sigma) \right) \right)$ and

$\tilde{z}_2 = \left(\left(\mathfrak{t}_{\tilde{z}_{r2}}(\sigma), \mathfrak{t}_{\tilde{z}_{i2}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{r2}}(\sigma), \mathfrak{l}_{\tilde{z}_{i2}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{r2}}(\sigma), \mathfrak{f}_{\tilde{z}_{i2}}(\sigma) \right) \right)$ be the two complex single-valued neutrosophic numbers (CSVNNs) and $\mu > 0$ be a real number. Then,

$$\tilde{z}_1 \oplus \tilde{z}_2 = \left(\begin{array}{c} \left(\mathfrak{t}_{\tilde{z}_{r1}}(\sigma) + \mathfrak{t}_{\tilde{z}_{r2}}(\sigma) - \mathfrak{t}_{\tilde{z}_{r1}}(\sigma)\mathfrak{t}_{\tilde{z}_{r2}}(\sigma), \right. \\ \left. \mathfrak{t}_{\tilde{z}_{i1}}(\sigma) + \mathfrak{t}_{\tilde{z}_{i2}}(\sigma) - \mathfrak{t}_{\tilde{z}_{i1}}(\sigma)\mathfrak{t}_{\tilde{z}_{i2}}(\sigma) \right), \\ \left(\mathfrak{l}_{\tilde{z}_{r1}}(\sigma)\mathfrak{l}_{\tilde{z}_{r2}}(\sigma), \mathfrak{l}_{\tilde{z}_{i1}}(\sigma)\mathfrak{l}_{\tilde{z}_{i2}}(\sigma) \right), \\ \left(\mathfrak{f}_{\tilde{z}_{r1}}(\sigma)\mathfrak{f}_{\tilde{z}_{r2}}(\sigma), \mathfrak{f}_{\tilde{z}_{i1}}(\sigma)\mathfrak{f}_{\tilde{z}_{i2}}(\sigma) \right) \end{array} \right) \quad (3)$$

$$\tilde{z}_1 \otimes \tilde{z}_2 = \left(\begin{array}{c} \left(\mathfrak{t}_{\tilde{z}_{r1}}(\sigma)\mathfrak{t}_{\tilde{z}_{r2}}(\sigma), \mathfrak{t}_{\tilde{z}_{i1}}(\sigma)\mathfrak{t}_{\tilde{z}_{i2}}(\sigma) \right), \\ \left(\mathfrak{l}_{\tilde{z}_{r1}}(\sigma) + \mathfrak{l}_{\tilde{z}_{r2}}(\sigma) - \mathfrak{l}_{\tilde{z}_{r1}}(\sigma)\mathfrak{l}_{\tilde{z}_{r2}}(\sigma), \right. \\ \left. \mathfrak{l}_{\tilde{z}_{i1}}(\sigma) + \mathfrak{l}_{\tilde{z}_{i2}}(\sigma) - \mathfrak{l}_{\tilde{z}_{i1}}(\sigma)\mathfrak{l}_{\tilde{z}_{i2}}(\sigma) \right), \\ \left(\mathfrak{f}_{\tilde{z}_{r1}}(\sigma) + \mathfrak{f}_{\tilde{z}_{r2}}(\sigma) - \mathfrak{f}_{\tilde{z}_{r1}}(\sigma)\mathfrak{f}_{\tilde{z}_{r2}}(\sigma), \right. \\ \left. \mathfrak{f}_{\tilde{z}_{i1}}(\sigma) + \mathfrak{f}_{\tilde{z}_{i2}}(\sigma) - \mathfrak{f}_{\tilde{z}_{i1}}(\sigma)\mathfrak{f}_{\tilde{z}_{i2}}(\sigma) \right) \end{array} \right) \quad (4)$$

$$\mu \tilde{z}_1 = \left(\begin{array}{c} \left(1 - \left(1 - \left(\mathfrak{t}_{\tilde{z}_{r1}}(\sigma) \right)^\mu \right), 1 - \left(1 - \left(\mathfrak{t}_{\tilde{z}_{i1}}(\sigma) \right)^\mu \right) \right), \\ \left(\left(\mathfrak{l}_{\tilde{z}_{r1}}(\sigma) \right)^\mu, \left(\mathfrak{l}_{\tilde{z}_{i1}}(\sigma) \right)^\mu, \left(\mathfrak{f}_{\tilde{z}_{r1}}(\sigma) \right)^\mu, \left(\mathfrak{f}_{\tilde{z}_{i1}}(\sigma) \right)^\mu \right) \end{array} \right) \quad (5)$$

$$\tilde{z}_1^\mu = \left(\begin{array}{c} \left(\mathfrak{t}_{\tilde{z}_{r1}}(\sigma) \right)^\mu, \left(\mathfrak{t}_{\tilde{z}_{i1}}(\sigma) \right)^\mu, \\ \left(1 - \left(1 - \mathfrak{l}_{\tilde{z}_{r1}}(\sigma) \right)^\mu, 1 - \left(1 - \mathfrak{l}_{\tilde{z}_{i1}}(\sigma) \right)^\mu \right), \\ \left(1 - \left(1 - \mathfrak{f}_{\tilde{z}_{r1}}(\sigma) \right)^\mu, 1 - \left(1 - \mathfrak{f}_{\tilde{z}_{i1}}(\sigma) \right)^\mu \right) \end{array} \right) \quad (6)$$

Definition 4: [6] Consider $\tilde{z} = \langle \mathfrak{t}_{\tilde{z}}(\sigma), \mathfrak{l}_{\tilde{z}}(\sigma), \mathfrak{f}_{\tilde{z}}(\sigma) \rangle$ be a CSVNN over a set U , such that $\mathfrak{t}_{\tilde{z}}(\sigma) = \left(\mathfrak{t}_{\tilde{z}_r}(\sigma), \mathfrak{t}_{\tilde{z}_i}(\sigma) \right)$, $\mathfrak{l}_{\tilde{z}}(\sigma) = \left(\mathfrak{l}_{\tilde{z}_r}(\sigma), \mathfrak{l}_{\tilde{z}_i}(\sigma) \right)$, $\mathfrak{f}_{\tilde{z}}(\sigma) = \left(\mathfrak{f}_{\tilde{z}_r}(\sigma), \mathfrak{f}_{\tilde{z}_i}(\sigma) \right)$, then a score function $\mathcal{S}$ of $\tilde{z}$ is defined as

$$\mathcal{S}(\tilde{z}) = \frac{1}{2} \left(\frac{\mathfrak{t}_{\tilde{z}_r}(\sigma) + 1 - \mathfrak{l}_{\tilde{z}_r}(\sigma) + 1 - \mathfrak{f}_{\tilde{z}_r}(\sigma)}{3} + \frac{\mathfrak{t}_{\tilde{z}_i}(\sigma) + 1 - \mathfrak{l}_{\tilde{z}_i}(\sigma) + 1 - \mathfrak{f}_{\tilde{z}_i}(\sigma)}{3} \right). \quad (7)$$

Definition 5: [6] Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{rj}}(\sigma), \mathfrak{l}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right)$; ($j = 1, 2, \dots, n$), be the collection of CSVNNs, then the prioritized weighted aggregation operators are defined as;

$$CSVNPWA(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\frac{\gamma_1}{\sum_{j=1}^n \gamma_j} \tilde{z}_1 \oplus \frac{\gamma_2}{\sum_{j=1}^n \gamma_j} \tilde{z}_2 \oplus \dots \oplus \frac{\gamma_n}{\sum_{j=1}^n \gamma_j} \tilde{z}_n \right). \quad (8)$$

Then CSVNPWA is called CSVN prioritized weighted average operator, where $\gamma_j = \prod_{j=2}^n \mathcal{S}(\tilde{z}_j)$; $\gamma_1 = 1$ and $\mathcal{S}(\tilde{z}_j)$ is the score value of CSVNN $\tilde{z}_n$.

$$CSVNPWG(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\tilde{z}_1^{\frac{\gamma_1}{\sum_{j=1}^n \gamma_j}} \otimes \tilde{z}_2^{\frac{\gamma_2}{\sum_{j=1}^n \gamma_j}} \otimes \dots \otimes \tilde{z}_n^{\frac{\gamma_n}{\sum_{j=1}^n \gamma_j}} \right). \quad (9)$$

Then CSVNPWG is called CSVN prioritized weighted geometric operator, where $\gamma_j = \prod_{j=2}^n \mathcal{S}(\tilde{z}_j)$; $\gamma_1 = 1$ and $\mathcal{S}(\tilde{z}_j)$ are the score values of CSVNN $\tilde{z}_n$.

Theorem 1: [29] Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(I_{\tilde{z}_{rj}}(\sigma), I_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right); (j = 1, 2, \dots, n)$, be the collection of CSVNNs. Then we can also calculate the CSVNPWA by:

$$CSVNPWA(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\left(1 - \prod_{j=1}^n \left(1 - \mathfrak{t}_{\tilde{z}_{rj}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}, 1 - \prod_{j=1}^n \left(1 - \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} \right), \right. \\ \left. \left(\prod_{j=1}^n \left(I_{\tilde{z}_{rj}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}, \prod_{j=1}^n \left(I_{\tilde{z}_{ij}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} \right), \right. \\ \left. \left(\prod_{j=1}^n \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}, \prod_{j=1}^n \left(\mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} \right) \right) \quad (10)$$

Theorem 2: [29] Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(I_{\tilde{z}_{rj}}(\sigma), I_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right); (j = 1, 2, \dots, n)$, be the collection of CSVNNs. Then we can also calculate the CSVNPWG by
 $CSVNPGA(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n)$

$$= \left(\left(\prod_{j=1}^n \left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}, \prod_{j=1}^n \left(\mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} \right), \right. \\ \left. \left(1 - \prod_{j=1}^n \left(1 - \left(I_{\tilde{z}_{rj}}(\sigma) \right) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}, 1 - \prod_{j=1}^n \left(1 - \left(I_{\tilde{z}_{ij}}(\sigma) \right) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} \right) \right) \\ \left(\left(1 - \prod_{j=1}^n \left(1 - \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma) \right) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}, 1 - \prod_{j=1}^n \left(1 - \left(\mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} \right) \right) \quad (11)$$

Definition 6: [30] Consider two CSVNNs and $\mu \geq 0$ be a real number such that $\tilde{z}_1 = \left(\left(\mathfrak{t}_{\tilde{z}_{r1}}(\sigma), \mathfrak{t}_{\tilde{z}_{i1}}(\sigma) \right), \left(I_{\tilde{z}_{r1}}(\sigma), I_{\tilde{z}_{i1}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{r1}}(\sigma), \mathfrak{f}_{\tilde{z}_{i1}}(\sigma) \right) \right)$ and $\tilde{z}_2 = \left(\left(\mathfrak{t}_{\tilde{z}_{r2}}(\sigma), \mathfrak{t}_{\tilde{z}_{i2}}(\sigma) \right), \left(I_{\tilde{z}_{r2}}(\sigma), I_{\tilde{z}_{i2}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{r2}}(\sigma), \mathfrak{f}_{\tilde{z}_{i2}}(\sigma) \right) \right)$. Then,

$$\tilde{z}_1 \oplus \tilde{z}_2 = \left(\left(\frac{\mathfrak{t}_{\tilde{z}_{r1}}(\sigma) \cdot_e \mathfrak{t}_{\tilde{z}_{r2}}(\sigma)}{1 + \left(1 - \mathfrak{t}_{\tilde{z}_{r1}}(\sigma) \right) \cdot_e \left(1 - \mathfrak{t}_{\tilde{z}_{r2}}(\sigma) \right)}, \frac{\mathfrak{t}_{\tilde{z}_{i1}}(\sigma) \cdot_e \mathfrak{t}_{\tilde{z}_{i2}}(\sigma)}{1 + \left(1 - \mathfrak{t}_{\tilde{z}_{i1}}(\sigma) \right) \cdot_e \left(1 - \mathfrak{t}_{\tilde{z}_{i2}}(\sigma) \right)} \right), \right. \\ \left. \left(\frac{I_{\tilde{z}_{r1}}(\sigma) + I_{\tilde{z}_{r2}}(\sigma)}{1 + I_{\tilde{z}_{r1}}(\sigma) \cdot_e I_{\tilde{z}_{r2}}(\sigma)}, \frac{I_{\tilde{z}_{i1}}(\sigma) + I_{\tilde{z}_{i2}}(\sigma)}{1 + I_{\tilde{z}_{i1}}(\sigma) \cdot_e I_{\tilde{z}_{i2}}(\sigma)} \right), \right. \\ \left. \left(\frac{\mathfrak{f}_{\tilde{z}_{r1}}(\sigma) + \mathfrak{f}_{\tilde{z}_{r2}}(\sigma)}{1 + \mathfrak{f}_{\tilde{z}_{r1}}(\sigma) \cdot_e \mathfrak{f}_{\tilde{z}_{r2}}(\sigma)}, \frac{\mathfrak{f}_{\tilde{z}_{i1}}(\sigma) + \mathfrak{f}_{\tilde{z}_{i2}}(\sigma)}{1 + \mathfrak{f}_{\tilde{z}_{i1}}(\sigma) \cdot_e \mathfrak{f}_{\tilde{z}_{i2}}(\sigma)} \right) \right) \quad (12)$$

$$\tilde{z}_1 \otimes \tilde{z}_2 = \left(\left(\frac{\tilde{t}_{\tilde{z}_{r1}}(\sigma) + \tilde{t}_{\tilde{z}_{r2}}(\sigma)}{1 + \tilde{t}_{\tilde{z}_{r1}}(\sigma) \cdot_e \tilde{t}_{\tilde{z}_{r2}}(\sigma)} \right), \left(\frac{\tilde{I}_{\tilde{z}_{r1}}(\sigma) \cdot_e \tilde{I}_{\tilde{z}_{r2}}(\sigma)}{1 + (1 - \tilde{I}_{\tilde{z}_{r1}}(\sigma)) \cdot_e (1 - \tilde{I}_{\tilde{z}_{r2}}(\sigma))} \right), \right. \\ \left. \left(\frac{\tilde{t}_{\tilde{z}_{i1}}(\sigma) + \tilde{t}_{\tilde{z}_{i2}}(\sigma)}{1 + \tilde{t}_{\tilde{z}_{i1}}(\sigma) \cdot_e \tilde{t}_{\tilde{z}_{i2}}(\sigma)} \right), \left(\frac{\tilde{I}_{\tilde{z}_{i1}}(\sigma) \cdot_e \tilde{I}_{\tilde{z}_{i2}}(\sigma)}{1 + (1 - \tilde{I}_{\tilde{z}_{i1}}(\sigma)) \cdot_e (1 - \tilde{I}_{\tilde{z}_{i2}}(\sigma))} \right) \right), \quad (13)$$

$$\mu_{\tilde{z}_1}^{\tilde{z}} = \left(\left(\frac{(1 + \tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu} - (1 - \tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu}}{(1 + \tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu} + (1 - \tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu}} \right), \left(\frac{2(\tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu}}{(2 - \tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu} + (\tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu}} \right), \right. \\ \left(\frac{(1 + \tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu} - (1 - \tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu}}{(1 + \tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu} + (1 - \tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu}} \right), \left(\frac{2(\tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu}}{(2 - \tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu} + (\tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu}} \right), \\ \left(\frac{2(\tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu}}{(2 - \tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu} + (\tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu}} \right), \\ \left. \left(\frac{2(\tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu}}{(2 - \tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu} + (\tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu}} \right) \right), \quad (14)$$

$$\tilde{z}_1^{\mu} = \left(\left(\frac{2(\tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu}}{(2 - \tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu} + (\tilde{t}_{\tilde{z}_{r1}}(\sigma))^{\mu}} \right), \left(\frac{(1 + \tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu} - (1 - \tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu}}{(1 + \tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu} + (1 - \tilde{I}_{\tilde{z}_{r1}}(\sigma))^{\mu}} \right), \right. \\ \left(\frac{2(\tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu}}{(2 - \tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu} + (\tilde{t}_{\tilde{z}_{i1}}(\sigma))^{\mu}} \right), \left(\frac{(1 + \tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu} - (1 - \tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu}}{(1 + \tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu} + (1 - \tilde{I}_{\tilde{z}_{i1}}(\sigma))^{\mu}} \right), \\ \left(\frac{(1 + \tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu} - (1 - \tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu}}{(1 + \tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu} + (1 - \tilde{f}_{\tilde{z}_{r1}}(\sigma))^{\mu}} \right), \\ \left. \left(\frac{(1 + \tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu} - (1 - \tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu}}{(1 + \tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu} + (1 - \tilde{f}_{\tilde{z}_{i1}}(\sigma))^{\mu}} \right) \right), \quad (15)$$

3. Complex Single-Valued Neutrosophic Einstein Prioritized Aggregation Operators

Here, we discussed the complex single-valued neutrosophic Einstein prioritized aggregation operator (CSVNEPAO) and defined some properties.

3.1. CSVNEPWA Operator

In this section, we'll propose the CSVNEPWA operator and define some theorems and their properties.

Definition 7: Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{rj}}(\sigma), \mathfrak{l}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right); (j = 1, 2, \dots, n)$, be the collection of CSVNNs. Then CSVNEPWA be an n-dimensional mapping given by:

$$CSVNEPWA(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\frac{\gamma_1}{\sum_{j=1}^n \gamma_j} \cdot \tilde{z}_1 \oplus \frac{\gamma_2}{\sum_{j=1}^n \gamma_j} \cdot \tilde{z}_2 \oplus \dots \oplus \frac{\gamma_j}{\sum_{j=1}^n \gamma_j} \cdot \tilde{z}_j \right) \quad (16)$$

Where $\gamma_j = \prod_{j=2}^n \mathcal{S}(\tilde{z}_j)$; $\gamma_1 = 1$ and $\mathcal{S}(\tilde{z}_j)$ be the score value of CSVNN $\tilde{z}_j$.

Theorem 3: Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{rj}}(\sigma), \mathfrak{l}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right); (j = 1, 2, \dots, n)$, be the collection of CSVNNs, and to avoid complexity we write $\alpha = \mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \alpha' = \mathfrak{t}_{\tilde{z}_{ij}}(\sigma); \beta = \mathfrak{l}_{\tilde{z}_{rj}}(\sigma), \beta' = \mathfrak{l}_{\tilde{z}_{ij}}(\sigma); \gamma = \mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \gamma' = \mathfrak{f}_{\tilde{z}_{ij}}(\sigma)$. Then CSVNEPWA is given by;

$$CSVNEPWA(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\left(\frac{\prod_{j=1}^n (1 + \alpha)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} - \prod_{j=1}^n (1 - \alpha)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}{\prod_{j=1}^n (1 + \alpha)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} + \prod_{j=1}^n (1 - \alpha)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}, \frac{\prod_{j=1}^n (1 + \alpha')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} - \prod_{j=1}^n (1 - \alpha')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}{\prod_{j=1}^n (1 + \alpha')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} + \prod_{j=1}^n (1 - \alpha')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}, \right. \right. \quad (17)$$

$$\left. \left(\frac{2 \prod_{j=1}^n (\beta)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}{\prod_{j=1}^n (2 - \beta)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} + \prod_{j=1}^n (\beta)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}, \frac{2 \prod_{j=1}^n (\beta')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}{\prod_{j=1}^n (2 - \beta')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} + \prod_{j=1}^n (\beta')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}, \right. \right.$$

$$\left. \left(\frac{2 \prod_{j=1}^n (\gamma)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}{\prod_{j=1}^n (2 - \gamma)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} + \prod_{j=1}^n (\gamma)^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}, \frac{2 \prod_{j=1}^n (\gamma')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}}{\prod_{j=1}^n (2 - \gamma')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}} + \prod_{j=1}^n (\gamma')^{\frac{\gamma_j}{\sum_{j=1}^n \gamma_j}}} \right) \right)$$

Where $\gamma_j = \prod_{j=2}^n \mathcal{S}(\tilde{z}_j)$; $\gamma_1 = 1$ and $\mathcal{S}(\tilde{z}_j)$ are the score values of CSVNN.

Proof: See Appendix A.

3.2. CSVNEPWG Operator

Definition 8: Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{rj}}(\sigma), \mathfrak{l}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right); (j = 1, 2, \dots, n)$, be the collection of CSVNNs. Then CSVNEPWG be an n-dimensional mapping given by:

$$CSVNEPWG(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\tilde{z}_1^{\frac{Y_1}{\sum_{j=1}^n Y_j}} \otimes_e \tilde{z}_2^{\frac{Y_2}{\sum_{j=1}^n Y_j}} \otimes_e \dots \otimes_e \tilde{z}_n^{\frac{Y_n}{\sum_{j=1}^n Y_j}} \right), \quad (18)$$

where $Y_j = \prod_{j=2}^n \mathcal{S}(\tilde{z}_j)$; $Y_1 = 1$ and $\mathcal{S}(\tilde{z}_j)$ are the score values of CSVNN.

Theorem 8: Let $\tilde{z}_j = \left(\left(\mathfrak{t}_{\tilde{z}_{rj}}(\sigma), \mathfrak{t}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{l}_{\tilde{z}_{rj}}(\sigma), \mathfrak{l}_{\tilde{z}_{ij}}(\sigma) \right), \left(\mathfrak{f}_{\tilde{z}_{rj}}(\sigma), \mathfrak{f}_{\tilde{z}_{ij}}(\sigma) \right) \right)$; ($j = 1, 2, \dots, n$), be the collection of CSVNNs. Then we can also evaluate the CSVNEPWG by

$$CSVNEPWG(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n) = \left(\left(\frac{2 \prod_{j=1}^n \left(\mathfrak{t}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}{\prod_{j=1}^n \left(2 - \mathfrak{t}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} + \prod_{j=1}^n \left(\mathfrak{t}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}, \frac{2 \prod_{j=1}^n \left(\mathfrak{t}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}{\prod_{j=1}^n \left(2 - \mathfrak{t}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} + \prod_{j=1}^n \left(\mathfrak{t}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}} \right), \quad (19)$$

$$= \left(\left(\frac{\prod_{j=1}^n \left(1 + \mathfrak{l}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} - \prod_{j=1}^n \left(1 - \mathfrak{l}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}{\prod_{j=1}^n \left(1 + \mathfrak{l}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} + \prod_{j=1}^n \left(1 - \mathfrak{l}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}, \frac{\prod_{j=1}^n \left(1 + \mathfrak{l}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} - \prod_{j=1}^n \left(1 - \mathfrak{l}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}{\prod_{j=1}^n \left(1 + \mathfrak{l}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} + \prod_{j=1}^n \left(1 - \mathfrak{l}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}} \right)$$

$$\left(\left(\frac{\prod_{j=1}^n \left(1 + \mathfrak{f}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} - \prod_{j=1}^n \left(1 - \mathfrak{f}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}{\prod_{j=1}^n \left(1 + \mathfrak{f}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} + \prod_{j=1}^n \left(1 - \mathfrak{f}_{\tilde{z}_r}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}, \frac{\prod_{j=1}^n \left(1 + \mathfrak{f}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} - \prod_{j=1}^n \left(1 - \mathfrak{f}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}}{\prod_{j=1}^n \left(1 + \mathfrak{f}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}} + \prod_{j=1}^n \left(1 - \mathfrak{f}_{\tilde{z}_i}(\sigma) \right)^{\frac{Y_j}{\sum_{j=1}^n Y_j}}} \right)$$

Where $Y_j = \prod_{j=2}^n \mathcal{S}(\tilde{z}_j)$; $Y_1 = 1$ and $\mathcal{S}(\tilde{z}_j)$ are the score values of CSVNN.

Proof: See Appendix B.

4. Proposed methodology

In this section, we present the MCGDM strategy for handling the DM issue under the CSVNN framework by implementing the proposed operators. Moreover, a real-world case has also been discussed to demonstrate it.

4.1 . Algorithm

For solving the MCGDM problem, the following steps should be followed:

- 1) The value assigned by the DM is stored in decision matrix form.
- 2) All attributive values are of the same type; otherwise, they must be normalized.
- 3) Evaluate $\wp_{ij}^{(m)}$ by Eq. (23) of each decision matrix.
- 4) Calculate the aggregate matrix by using the proposed operators.
- 5) Compute $\wp_{ij}^{(m)}$ of the aggregated matrix.
- 6) Determine the CSVN values F_{ij} correspond to each alternative p_i by proposed operators.
- 7) Compute the score value of the aggregated alternatives and rank them.

4.2. Methodology

Consider that an MCGDM problem is evaluated by the DM, which comprises attributive values $C = \{C_1, C_2, \dots, C_n\}$ and alternatives $p = \{p_1, p_2, \dots, p_m\}$. The DMs determine the results correspond to each alternative under the CSVNN environment, and the value assigned by the DM is defined in the form of a decision matrix $D^{(m)} = (\mathcal{D}_{ij}^{(m)})_{m \times n}$ as.

$$D^{(m)} = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{matrix} & \begin{bmatrix} \mathcal{Z}_{11}^{(m)} & \mathcal{Z}_{12}^{(m)} & \dots & \mathcal{Z}_{1n}^{(m)} \\ \mathcal{Z}_{21}^{(m)} & \mathcal{Z}_{22}^{(m)} & \dots & \mathcal{Z}_{2n}^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{Z}_{m1}^{(m)} & \mathcal{Z}_{m2}^{(m)} & \dots & \mathcal{Z}_{mn}^{(m)} \end{bmatrix} \end{matrix}$$

The approach to finding the best alternative can be summed up as follows:

- 1) If the attributive values are of the same type, then it doesn't require normalization. However, if it belongs to different types of attributive values (profit(t_p)/ cost(t_c) type criteria), then these values must be normalized from $D^{(m)}$ into $K^{(m)} = (\mathcal{M}_{ij}^{(m)})_{m \times n}$, where $(\mathcal{M}_{ij}^{(m)})_{m \times n}$ is evaluated by

$$(\mathcal{M}_{ij}^{(m)})_{m \times n} = \begin{cases} (\mathcal{D}_{ij}^{(m)})^c; j \in t_c \\ \mathcal{D}_{ij}^{(m)}; j \in t_p \end{cases} \quad (20)$$

$(\mathcal{D}_{ij}^{(m)})^c$ represents the complement of $\mathcal{D}_{ij}^{(m)}$

- 2) Evaluate $\wp_{ij}^{(m)}$ by using the formula.

$$\wp_{ij}^{(m)} = \prod_{k=1}^{m-1} \otimes (\mathcal{M}_{ij}^{(k)}) \quad (m = 2, \dots, n), \quad \wp_{ij}^{(1)} = 1 \quad (21)$$

- 3) Use the aggregate operator AOs to calculate the aggregate matrix $M_{ij}^{(m)} = (F_{ij}^{(m)})_{m \times n}$ by aggregating all the decision matrices $K^{(m)} = (\mathcal{M}_{ij}^{(m)})_{m \times n}$

$$F_{ij} = \text{CSVNEPWA}(\mathcal{M}_{ij}^{(1)}, \mathcal{M}_{ij}^{(2)}, \dots, \mathcal{M}_{ij}^{(m)}) =$$

$$\begin{aligned}
 & \left(\frac{\prod_{k=1}^m \left(1 + \mathfrak{t}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} - \prod_{j=1}^m \left(1 - \mathfrak{t}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}{\prod_{j=1}^m \left(1 + \mathfrak{t}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} + \prod_{j=1}^m \left(1 - \mathfrak{t}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}, \right. \\
 & \left. \frac{\prod_{j=1}^m \left(1 + \mathfrak{t}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} - \prod_{j=1}^m \left(1 - \mathfrak{t}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}{\prod_{j=1}^m \left(1 + \mathfrak{t}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} + \prod_{j=1}^m \left(1 - \mathfrak{t}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}} \right), \tag{22} \\
 = & \left(\frac{2 \prod_{j=1}^m \left(\mathfrak{l}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}{\prod_{j=1}^m \left(2 - \mathfrak{l}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} + \prod_{j=1}^m \left(\mathfrak{l}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}, \right. \\
 & \left. \frac{2 \prod_{j=1}^m \left(\mathfrak{l}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}{\prod_{j=1}^m \left(2 - \mathfrak{l}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} + \prod_{j=1}^m \left(\mathfrak{l}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}} \right), \\
 & \left(\frac{2 \prod_{j=1}^m \left(\mathfrak{f}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}{\prod_{j=1}^m \left(2 - \mathfrak{f}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} + \prod_{j=1}^m \left(\mathfrak{f}_{3rij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}, \right. \\
 & \left. \frac{2 \prod_{j=1}^m \left(\mathfrak{f}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}}{\prod_{j=1}^m \left(2 - \mathfrak{f}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}} + \prod_{j=1}^m \left(\mathfrak{f}_{3ij}(\sigma) \right)^{\frac{\gamma_k}{\sum_{k=1}^m \gamma_k}}} \right)
 \end{aligned}$$

or

$$F_{ij} = \text{CSVNEPWG}(\mathcal{M}_{ij}^{(1)}, \mathcal{M}_{ij}^{(2)}, \dots, \mathcal{M}_{ij}^{(m)})$$

$$\begin{aligned}
 & \left(\begin{aligned} & \frac{2 \prod_{j=1}^n \left(t_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(2 - t_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(t_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}, \\ & \frac{2 \prod_{j=1}^n \left(t_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(2 - t_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(t_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}} \end{aligned} \right), \\
 = & \left(\begin{aligned} & \frac{\prod_{j=1}^n \left(1 + I_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} - \prod_{j=1}^n \left(1 - I_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(1 + I_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(1 - I_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}, \\ & \frac{\prod_{j=1}^n \left(1 + I_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} - \prod_{j=1}^n \left(1 - I_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(1 + I_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(1 - I_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}, \\ & \frac{\prod_{j=1}^n \left(1 + f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} - \prod_{j=1}^n \left(1 - f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(1 + f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(1 - f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}, \\ & \frac{\prod_{j=1}^n \left(1 + f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} - \prod_{j=1}^n \left(1 - f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(1 + f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(1 - f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}} \end{aligned} \right), \\
 & \left(\begin{aligned} & \frac{\prod_{j=1}^n \left(1 + f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} - \prod_{j=1}^n \left(1 - f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(1 + f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(1 - f_{z_{r ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}, \\ & \frac{\prod_{j=1}^n \left(1 + f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} - \prod_{j=1}^n \left(1 - f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}}{\prod_{j=1}^n \left(1 + f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}} + \prod_{j=1}^n \left(1 - f_{z_{i ij}}(\sigma) \right)^{\frac{Y_k}{\sum_{k=1}^m Y_k}}} \end{aligned} \right)
 \end{aligned} \tag{23}$$

4) Compute $\wp_{ij}^{(m)}$ by using the formula.

$$\wp_{ij}^{(m)} = \prod_{k=1}^{m-1} \otimes (F_{ik}) \quad (m = 2, \dots, n), \quad \wp_{i1} = 1 \tag{24}$$

5) Determine the CSVN values F_{ij} correspond to each alternative p_i by using the CSVNEPWA (or CSVNEPWG) operator formula defined in Eqs. (24) and (25), respectively.

6) Calculate the score value of the aggregated alternatives and rank them in descending order. So, the optimal selection was made.

We present a more realistic assessment of the quality of a variety of shelter services in the public Metrobus system. For the analysis of shelter service quality of the public Metrobus system, multiple criteria C_i are used, and several human experts are hired to support them. Consider $C_i = \{C_1, C_2, C_3, C_4, \text{and } C_5\}$ be the set of criteria which consists of Weather Protection, Seating and Comfort, Cleanliness and Maintenance, Information Systems, and Safety and Lighting.

$$p_i = \begin{cases} p_1 \text{ (Enclosed Lounge),} \\ p_2 \text{ (Extended Canopy, Glass),} \\ p_3 \text{ (Modular Green Shelter),} \\ p_4 \text{ (Smart Digital Shelter),} \\ p_5 \text{ (Current Standard Shelter)} \end{cases}$$

4.2.1. Evaluation by CSVNEPWA operator

1) The value assigned by the DM is defined in the form of a decision matrix $\mathcal{D}^{(m)} = (\mathcal{D}_{ij}^{(m)})_{m \times n}$ as shown in Tables 1-3.

(All these values assigned by the DM are hypothetical, and they are restricted to assigning these values in the form of a CSVN fuzzy number to each alternative).

Table 1

Decision_matrix by expert one ($\mathcal{D}^{(1)}$)

	C_1	C_2	C_3	C_4	C_5
p_1	$\begin{pmatrix} (0.75,0.36), \\ (0.22,0.30), \\ (0.44,0.55) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.20), \\ (0.75,0.15), \\ (0.45,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.55,0.34), \\ (0.18,0.44), \\ (0.50,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.80,0.25), \\ (0.60,0.35), \\ (0.70,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.25), \\ (0.75,0.20), \\ (0.40,0.35) \end{pmatrix}$
p_2	$\begin{pmatrix} (0.40,0.13), \\ (0.55,0.15), \\ (0.20,0.60) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.80), \\ (0.21,0.42), \\ (0.18,0.27) \end{pmatrix}$	$\begin{pmatrix} (0.45,0.63), \\ (0.55,0.30), \\ (0.90,0.50) \end{pmatrix}$	$\begin{pmatrix} (0.65,0.55), \\ (0.25,0.35), \\ (0.70,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.75), \\ (0.45,0.55), \\ (0.80,0.65) \end{pmatrix}$
p_3	$\begin{pmatrix} (0.20,0.30), \\ (0.40,0.50), \\ (0.60,0.70) \end{pmatrix}$	$\begin{pmatrix} (0.15,0.20), \\ (0.25,0.30), \\ (0.45,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.80,0.20), \\ (0.70,0.30), \\ (0.60,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.65,0.35), \\ (0.55,0.25), \\ (0.40,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.20,0.40), \\ (0.10,0.25), \\ (0.45,0.54) \end{pmatrix}$
p_4	$\begin{pmatrix} (0.10,0.20), \\ (0.15,0.25), \\ (0.10,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.55,0.50), \\ (0.65,0.60), \\ (0.30,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.40,0.45), \\ (0.35,0.30), \\ (0.70,0.20) \end{pmatrix}$	$\begin{pmatrix} (0.20,0.10), \\ (0.35,0.55), \\ (0.45,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.15,0.44), \\ (0.65,0.70), \\ (0.26,0.60) \end{pmatrix}$
p_5	$\begin{pmatrix} (0.55,0.45), \\ (0.20,0.20), \\ (0.70,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.30,0.50), \\ (0.60,0.40), \\ (0.35,0.20) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.55), \\ (0.45,0.45), \\ (0.30,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.45,0.50), \\ (0.65,0.70), \\ (0.30,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.45), \\ (0.55,0.60), \\ (0.30,0.45) \end{pmatrix}$

Table 2

Decision matrix by expert two ($\mathcal{D}^{(2)}$)

	C_1	C_2	C_3	C_4	C_5
p_1	$\begin{pmatrix} (0.45,0.70), \\ (0.35,0.30), \\ (0.55,0.55) \end{pmatrix}$	$\begin{pmatrix} (0.50,0.25), \\ (0.45,0.10), \\ (0.55,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.50,0.55), \\ (0.20,0.45), \\ (0.40,0.50) \end{pmatrix}$	$\begin{pmatrix} (0.30,0.30), \\ (0.40,0.35), \\ (0.70,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.50,0.35), \\ (0.70,0.50), \\ (0.30,0.40) \end{pmatrix}$
p_2	$\begin{pmatrix} (0.40,0.15), \\ (0.50,0.20), \\ (0.35,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.20,0.60), \\ (0.10,0.45), \\ (0.15,0.20) \end{pmatrix}$	$\begin{pmatrix} (0.35,0.65), \\ (0.05,0.40), \\ (0.20,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.65), \\ (0.50,0.45), \\ (0.50,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.65,0.75), \\ (0.45,0.55), \\ (0.40,0.35) \end{pmatrix}$
p_3	$\begin{pmatrix} (0.30,0.20), \\ (0.50,0.50), \\ (0.70,0.60) \end{pmatrix}$	$\begin{pmatrix} (0.35,0.70), \\ (0.65,0.30), \\ (0.55,0.80) \end{pmatrix}$	$\begin{pmatrix} (0.80,0.10), \\ (0.60,0.30), \\ (0.60,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.65,0.45), \\ (0.45,0.20), \\ (0.40,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.40), \\ (0.60,0.20), \\ (0.45,0.25) \end{pmatrix}$
p_4	$\begin{pmatrix} (0.20,0.20), \\ (0.15,0.20), \\ (0.35,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.45,0.40), \\ (0.50,0.30), \\ (0.30,0.55) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.45), \\ (0.30,0.30), \\ (0.40,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.10,0.10), \\ (0.55,0.45), \\ (0.35,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.15,0.40), \\ (0.45,0.70), \\ (0.20,0.65) \end{pmatrix}$
p_5	$\begin{pmatrix} (0.55,0.40), \\ (0.25,0.20), \\ (0.70,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.30,0.55), \\ (0.65,0.40), \\ (0.30,0.25) \end{pmatrix}$	$\begin{pmatrix} (0.20,0.55), \\ (0.40,0.40), \\ (0.35,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.45,0.50), \\ (0.60,0.75), \\ (0.30,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.40), \\ (0.50,0.65), \\ (0.35,0.40) \end{pmatrix}$

Table 3

Decision matrix by expert three ($\mathcal{D}^{(3)}$)

	C_1	C_2	C_3	C_4	C_5
p_1	$\begin{pmatrix} (0.35,0.70), \\ (0.55,0.30), \\ (0.45,0.50) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.20), \\ (0.35,0.10), \\ (0.45,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.50), \\ (0.10,0.40), \\ (0.30,0.50) \end{pmatrix}$	$\begin{pmatrix} (0.35,0.30), \\ (0.45,0.30), \\ (0.75,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.40,0.35), \\ (0.60,0.55), \\ (0.20,0.45) \end{pmatrix}$
p_2	$\begin{pmatrix} (0.30,0.15), \\ (0.40,0.25), \\ (0.65,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.20,0.60), \\ (0.10,0.45), \\ (0.15,0.20) \end{pmatrix}$	$\begin{pmatrix} (0.35,0.65), \\ (0.05,0.40), \\ (0.20,0.30) \end{pmatrix}$	$\begin{pmatrix} (0.60,0.65), \\ (0.50,0.45), \\ (0.50,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.65,0.75), \\ (0.45,0.55), \\ (0.40,0.35) \end{pmatrix}$
p_3	$\begin{pmatrix} (0.45,0.20), \\ (0.65,0.50), \\ (0.45,0.60) \end{pmatrix}$	$\begin{pmatrix} (0.45,0.70), \\ (0.55,0.30), \\ (0.65,0.80) \end{pmatrix}$	$\begin{pmatrix} (0.70,0.15), \\ (0.60,0.35), \\ (0.70,0.45) \end{pmatrix}$	$\begin{pmatrix} (0.65,0.40), \\ (0.35,0.20), \\ (0.25,0.40) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.10), \\ (0.40,0.35), \\ (0.65,0.50) \end{pmatrix}$
p_4	$\begin{pmatrix} (0.30,0.15), \\ (0.25,0.40), \\ (0.40,0.25) \end{pmatrix}$	$\begin{pmatrix} (0.45,0.40), \\ (0.45,0.60), \\ (0.25,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.50,0.40), \\ (0.20,0.30), \\ (0.30,0.20) \end{pmatrix}$	$\begin{pmatrix} (0.10,0.20), \\ (0.35,0.45), \\ (0.55,0.60) \end{pmatrix}$	$\begin{pmatrix} (0.25,0.45), \\ (0.25,0.75), \\ (0.25,0.45) \end{pmatrix}$
p_5	$\begin{pmatrix} (0.50,0.35), \\ (0.45,0.20), \\ (0.60,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.30,0.65), \\ (0.65,0.40), \\ (0.30,0.25) \end{pmatrix}$	$\begin{pmatrix} (0.40,0.55), \\ (0.60,0.30), \\ (0.85,0.20) \end{pmatrix}$	$\begin{pmatrix} (0.35,0.40), \\ (0.40,0.65), \\ (0.30,0.35) \end{pmatrix}$	$\begin{pmatrix} (0.30,0.50), \\ (0.50,0.60), \\ (0.60,0.30) \end{pmatrix}$

2) As there is no cost criterion, it doesn't require normalization.

3) In this step, we compute the prioritized weights by following Eq. (23) given in Step 2 of the algorithm.

$$\wp_{ij}^{(1)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\wp_{ij}^{(2)} = \begin{bmatrix} 0.6000 & 0.5167 & 0.5783 & 0.4916 & 0.5250 \\ 0.5050 & 0.6617 & 0.4716 & 0.5833 & 0.4833 \\ 0.3833 & 0.4917 & 0.5000 & 0.5583 & 0.5433 \\ 0.5833 & 0.5250 & 0.5500 & 0.4333 & 0.3966 \\ 0.5833 & 0.5417 & 0.5500 & 0.4750 & 0.4667 \end{bmatrix}$$

$$\wp_{ij}^{(3)} = \begin{bmatrix} 0.3400 & 0.2755 & 0.3373 & 0.2253 & 0.2581 \\ 0.2609 & 0.4300 & 0.3183 & 0.3256 & 0.2940 \\ 0.1405 & 0.2253 & 0.2500 & 0.3396 & 0.2852 \\ 0.3256 & 0.2800 & 0.3437 & 0.1841 & 0.1685 \\ 0.3256 & 0.2934 & 0.3025 & 0.2256 & 0.2138 \end{bmatrix}$$

4) Calculate the aggregation matrix $M_{ij}^{(m)} = \left(F_{ij}^{(m)}\right)_{m \times n}$ by aggregating all the decision matrices by using Eq. (24), which is shown in Table 4.

Table 4

Collective preference of all experts by aggregating the decision matrix 1, 2, and 3 using CSVNEPWA operator

	C_1	C_2	C_3	C_4	C_5
p_1	$\begin{pmatrix} (0.6784,0.7237), \\ (0.2166,0.1513), \\ (0.1631,0.0816) \end{pmatrix}$	$\begin{pmatrix} (0.6089,0.4800), \\ (0.4754,0.0652), \\ (0.1697,0.0569) \end{pmatrix}$	$\begin{pmatrix} (0.5782,0.6338), \\ (0.1633,0.02452), \\ (0.1555,0.0567) \end{pmatrix}$	$\begin{pmatrix} (0.6793,0.5327), \\ (0.3711,0.1680), \\ (0.2459,0.0560) \end{pmatrix}$	$\begin{pmatrix} (0.6322,0.5496), \\ (0.5749,0.1551), \\ (0.1042,0.0487) \end{pmatrix}$
p_2	$\begin{pmatrix} (0.2669,0.0813), \\ (0.6243,0.1403), \\ (0.2635,0.6188) \end{pmatrix}$	$\begin{pmatrix} (0.1455,0.4170), \\ (0.3747,0.2644), \\ (0.2350,0.4711) \end{pmatrix}$	$\begin{pmatrix} (0.2561,0.3854), \\ (0.3936,0.2210), \\ (0.3849,0.5698) \end{pmatrix}$	$\begin{pmatrix} (0.3689,0.3069), \\ (0.5615,0.2558), \\ (0.4518,0.6033) \end{pmatrix}$	$\begin{pmatrix} (0.3860,0.4607), \\ (0.6050,0.2981), \\ (0.4398,0.6439) \end{pmatrix}$
p_3	$\begin{pmatrix} (0.0917,0.1343), \\ (0.7936,0.6820), \\ (0.8287,0.8781) \end{pmatrix}$	$\begin{pmatrix} (0.1015,0.2310), \\ (0.7585,0.5763), \\ (0.7746,0.8238) \end{pmatrix}$	$\begin{pmatrix} (0.3376,0.1045), \\ (0.8579,0.5916), \\ (0.8229,0.7613) \end{pmatrix}$	$\begin{pmatrix} (0.2270,0.1753), \\ (0.8215,0.5744), \\ (0.7636,0.7903) \end{pmatrix}$	$\begin{pmatrix} (0.0816,0.1764), \\ (0.7254,0.5710), \\ (0.7818,0.7632) \end{pmatrix}$
p_4	$\begin{pmatrix} (0.0295,0.0544), \\ (0.8479,0.8489), \\ (0.8334,0.8429) \end{pmatrix}$	$\begin{pmatrix} (0.0983,0.1073), \\ (0.9132,0.8624), \\ (0.8229,0.8346) \end{pmatrix}$	$\begin{pmatrix} (0.1003,0.1056), \\ (0.8585,0.8300), \\ (0.8756,0.7873) \end{pmatrix}$	$\begin{pmatrix} (0.0374,0.0484), \\ (0.8829,0.8724), \\ (0.8444,0.8021) \end{pmatrix}$	$\begin{pmatrix} (0.0355,0.0981), \\ (0.9096,0.9145), \\ (0.8052,0.8735) \end{pmatrix}$
p_5	$\begin{pmatrix} (0.0616,0.0417), \\ (0.8643,0.9268), \\ (0.9745,0.9483) \end{pmatrix}$	$\begin{pmatrix} (0.0486,0.0610), \\ (0.8960,0.9383), \\ (0.9269,0.9047) \end{pmatrix}$	$\begin{pmatrix} (0.0435,0.0690), \\ (0.6890,0.9353), \\ (0.9208,0.9113) \end{pmatrix}$	$\begin{pmatrix} (0.0566,0.0512), \\ (0.9012,0.9754), \\ (0.9342,0.9466) \end{pmatrix}$	$\begin{pmatrix} (0.0389,0.0409), \\ (0.8966,0.9710), \\ (0.9440,0.9490) \end{pmatrix}$

5) Compute $\wp_{ij}^{(m)}$ by using Eq. (26)

$$\wp_{ij} = \begin{bmatrix} 1 & 0.7933 & 0.3572 & 0.0621 & 0.0074 \\ 1 & 0.7317 & 0.4146 & 0.1106 & 0.0201 \\ 1 & 0.8033 & 0.4124 & 0.0969 & 0.0141 \\ 1 & 0.7417 & 0.3585 & 0.0956 & 0.0378 \\ 1 & 0.6850 & 0.3311 & 0.0910 & 0.0109 \end{bmatrix}$$

6) Determine the CSVN values F_{ij} correspond to each alternative p_i by using Eq. (24) given in Table 5.

Table 5

CSVNN corresponds to each Alternative

p_1	p_2	p_3	p_4	p_5
$\begin{pmatrix} (0.9168, 0.8900), \\ (0.0395, 0.0013), \\ (0.0037, 0.0018) \end{pmatrix}$	$\begin{pmatrix} (0.3978, 0.4953), \\ (0.2451, 0.0441), \\ (0.1158, 0.3321) \end{pmatrix}$	$\begin{pmatrix} (0.1380, 0.1188), \\ (0.8047, 0.6164), \\ (0.7887, 0.8044) \end{pmatrix}$	$\begin{pmatrix} (0.0119, 0.0161), \\ (0.9715, 0.9657), \\ (0.8932, 0.8791) \end{pmatrix}$	$\begin{pmatrix} (0.0018, 0.0021), \\ (0.9930, 0.9962), \\ (0.9963, 0.9961) \end{pmatrix}$

7) Calculate the score value of the aggregated alternatives by using Eq. (7).

$$\mathcal{S}(p_1) = 0.9601; \mathcal{S}(p_2) = 0.6926; \mathcal{S}(p_3) = 0.2072; \mathcal{S}(p_4) = 0.0529; \mathcal{S}(p_5) = 0.0037$$

8) Rank them in descending order. i.e. $p_1 > p_2 > p_3 > p_4 > p_5$.

4.2.2. Evaluation by CSVNEPWG operator

- 1) The value assigned by the DM is defined in the form of a decision matrix $\mathcal{D}^{(m)} = (\mathcal{D}_{ij}^{(m)})_{m \times n}$ which is shown in Table 1-3.
- 2) As there is no cost criterion, it doesn't require normalization.
- 3) Evaluate $\wp_{ij}^{(m)}$ by using Eq. (23)

$$\wp_{ij}^{(1)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\wp_{ij}^{(2)} = \begin{bmatrix} 0.6000 & 0.5167 & 0.5783 & 0.4916 & 0.5250 \\ 0.5050 & 0.6617 & 0.4716 & 0.5833 & 0.4833 \\ 0.3833 & 0.4917 & 0.5000 & 0.5583 & 0.5433 \\ 0.5833 & 0.5250 & 0.5500 & 0.4333 & 0.3966 \\ 0.5833 & 0.5417 & 0.5500 & 0.4750 & 0.4667 \\ 0.3400 & 0.2755 & 0.3373 & 0.2253 & 0.2581 \\ 0.2609 & 0.4300 & 0.3183 & 0.3256 & 0.2940 \\ 0.1405 & 0.2253 & 0.2500 & 0.3396 & 0.2852 \\ 0.3256 & 0.2800 & 0.3437 & 0.1841 & 0.1685 \\ 0.3256 & 0.2934 & 0.3025 & 0.2256 & 0.2138 \end{bmatrix}$$

- 4) Calculate the aggregate matrix $M_{ij}^{(m)} = (F_{ij}^{(m)})_{m \times n}$ by aggregating all the decision matrices by using Eq. (25), which is given below (Table 6).

Table 6

Collective preference of all experts by aggregating the decision matrix 1, 2, and 3 using CSVNEPWG operator

	C_1	C_2	C_3	C_4	C_5
p_1	$\begin{pmatrix} (0.3207, 0.4112), \\ (0.5417, 0.4307), \\ (0.6536, 0.7089) \end{pmatrix}$	$\begin{pmatrix} (0.4295, 0.1007), \\ (0.6918, 0.1620), \\ (0.6272, 0.5300) \end{pmatrix}$	$\begin{pmatrix} (0.4270, 0.3217), \\ (0.2151, 0.5495), \\ (0.5230, 0.5508) \end{pmatrix}$	$\begin{pmatrix} (0.2617, 0.1196), \\ (0.6710, 0.4849), \\ (0.8806, 0.5898) \end{pmatrix}$	$\begin{pmatrix} (0.3148, 0.1458), \\ (0.8552, 0.5972), \\ (0.4392, 0.5657) \end{pmatrix}$
p_2	$\begin{pmatrix} (0.4381, 0.2046), \\ (0.4217, 0.1698), \\ (0.3603, 0.4245) \end{pmatrix}$	$\begin{pmatrix} (0.3360, 0.7398), \\ (0.1030, 0.3429), \\ (0.1212, 0.1697) \end{pmatrix}$	$\begin{pmatrix} (0.4800, 0.7097), \\ (0.1967, 0.2976), \\ (0.4762, 0.3048) \end{pmatrix}$	$\begin{pmatrix} (0.7184, 0.7042), \\ (0.3110, 0.3078), \\ (0.4394, 0.2946) \end{pmatrix}$	$\begin{pmatrix} (0.7150, 0.8084), \\ (0.3523, 0.4379), \\ (0.4632, 0.3685) \end{pmatrix}$
p_3	$\begin{pmatrix} (0.6524, 0.6006), \\ (0.2339, 0.2243), \\ (0.2559, 0.3260) \end{pmatrix}$	$\begin{pmatrix} (0.5910, 0.7265), \\ (0.2660, 0.1518), \\ (0.3009, 0.4085) \end{pmatrix}$	$\begin{pmatrix} (0.8834, 0.4468), \\ (0.3432, 0.1581), \\ (0.3504, 0.2122) \end{pmatrix}$	$\begin{pmatrix} (0.8519, 0.7226), \\ (0.1983, 0.0900), \\ (0.1493, 0.1796) \end{pmatrix}$	$\begin{pmatrix} (0.5919, 0.6071), \\ (0.1780, 0.1178), \\ (0.2466, 0.1963) \end{pmatrix}$
p_4	$\begin{pmatrix} (0.8173, 0.8181), \\ (0.0295, 0.0466), \\ (0.0466, 0.0496) \end{pmatrix}$	$\begin{pmatrix} (0.8662, 0.8498), \\ (0.1407, 0.1332), \\ (0.0682, 0.1043) \end{pmatrix}$	$\begin{pmatrix} (0.8733, 0.8502), \\ (0.0672, 0.0723), \\ (0.1185, 0.0565) \end{pmatrix}$	$\begin{pmatrix} (0.6798, 0.6817), \\ (0.1096, 0.1289), \\ (0.1203, 0.1127) \end{pmatrix}$	$\begin{pmatrix} (0.7588, 0.8642), \\ (0.1073, 0.1862), \\ (0.0501, 0.1380) \end{pmatrix}$
p_5	$\begin{pmatrix} (0.9551, 0.9385), \\ (0.0272, 0.01804), \\ (0.0723, 0.4032) \end{pmatrix}$	$\begin{pmatrix} (0.8924, 0.9421), \\ (0.0928, 0.0527), \\ (0.0408, 0.0296) \end{pmatrix}$	$\begin{pmatrix} (0.8694, 0.9371), \\ (0.0771, 0.0567), \\ (0.0976, 0.0386) \end{pmatrix}$	$\begin{pmatrix} (0.9316, 0.9391), \\ (0.0645, 0.0889), \\ (0.0316, 0.0455) \end{pmatrix}$	$\begin{pmatrix} (0.9141, 0.9429), \\ (0.0522, 0.0662), \\ (0.0421, 0.0370) \end{pmatrix}$

- 5) Compute $\wp_{ij}^{(m)}$ by using Eq. (26)

$$\wp_{ij} = \begin{bmatrix} 1 & 0.4017 & 0.2296 & 0.1607 & 0.1393 \\ 1 & 0.4533 & 0.3324 & 0.2327 & 0.2032 \\ 1 & 0.5333 & 0.3644 & 0.2611 & 0.2315 \\ 1 & 0.3333 & 0.2333 & 0.1987 & 0.1589 \\ 1 & 0.3666 & 0.2505 & 0.1920 & 0.1648 \end{bmatrix}$$

- 6) Determine the CSVN values F_{ij} correspond to each alternative p_i by using Eq. (25) given in Table 7.

Table 7

CSVNN corresponds to each Alternative

p_1	p_2	p_3	p_4	p_5
$\begin{pmatrix} (0.0340, 0.0061), \\ (0.9255, 0.7216), \\ (0.9389, 0.9032) \end{pmatrix}$	$\begin{pmatrix} (0.4792, 0.5764), \\ (0.2374, 0.2557), \\ (0.3198, 0.2638) \end{pmatrix}$	$\begin{pmatrix} (0.7477, 0.7145), \\ (0.1258, 0.0779), \\ (0.1666, 0.1547) \end{pmatrix}$	$\begin{pmatrix} (0.8666, 0.8768), \\ (0.0388, 0.0416), \\ (0.0358, 0.0617) \end{pmatrix}$	$\begin{pmatrix} (0.9379, 0.9572), \\ (0.0256, 0.0218), \\ (0.0232, 0.0410) \end{pmatrix}$

- 7) Calculate the score value of the aggregated alternatives by using Eq. (7), shown in Table 8.
 $\mathcal{S}(p_1) = 0.0938; \mathcal{S}(p_2) = 0.6333; \mathcal{S}(p_3) = 0.8050; \mathcal{S}(p_4) = 0.9066; \mathcal{S}(p_5) = 0.9500$
- 8) Rank them in descending order. $p_1 > p_2 > p_3 > p_4 > p_5$.

Table 8

Score value of Alternatives

	SCORE VALUE BY USING CSVNEPWA	SCORE VALUE BY USING CSVNEPWG
p_1	0.9601	0.0938
p_2	0.6926	0.6333
p_3	0.2072	0.8050
p_4	0.0529	0.9066
p_5	0.0037	0.9500

The pictorial representation of score values for each alternative using AOs is shown in Figure 1.

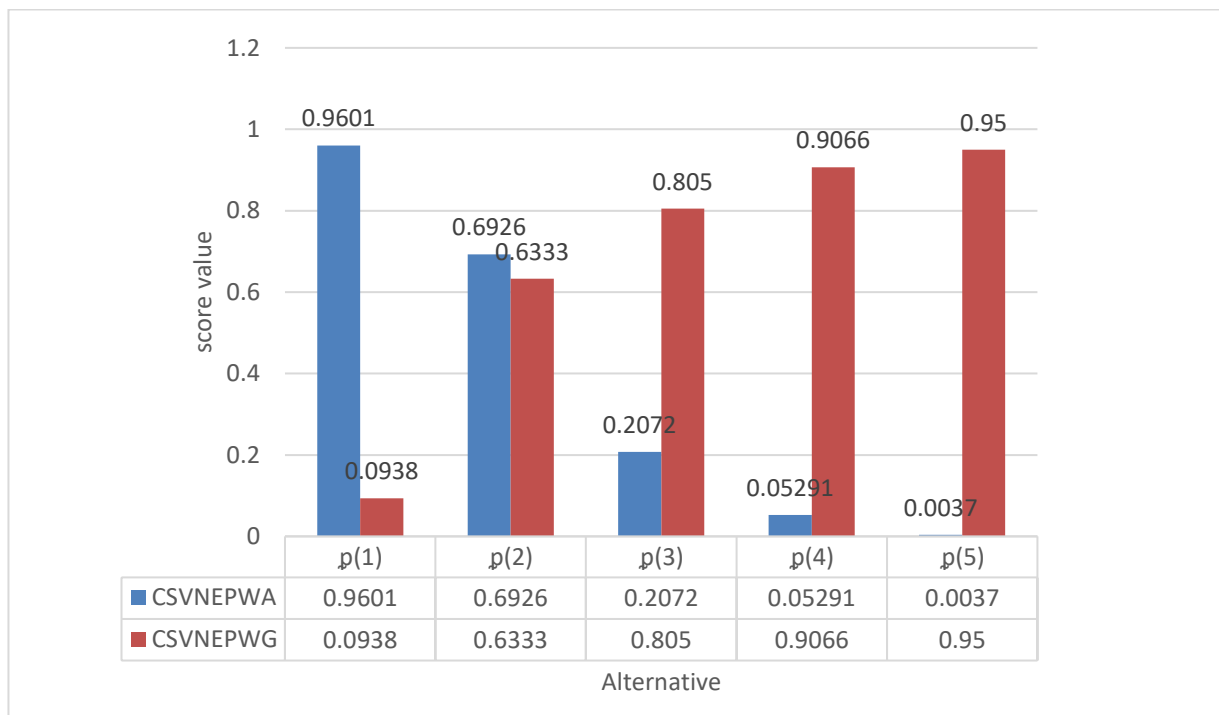


Fig. 1. Score Value of Alternative

4.3. Comparison Analysis

In this section, we examined prior techniques alongside the proposed technique, and the comparison reveals that the previous methods are not very effective, and some fail to demonstrate

the relationship between the attributive values. Zhang et al. [24] defined the SVNS, and Serrano et al. [32] discussed a simplified neutrosophic set that does not capture the relationship between the attributive values and doesn't address complex environments. Afterward, Zhang et al. [33] proposed the SVN Einstein aggregate operators, which enable handling the periodicity operation feature term, but they cannot handle the phase term. Moreover, it cannot show the relationship between the attributive values. Then, Raiz et al. [34] proposed that SVN prioritized AOs are more efficient than prior methodologies, but they cannot handle the phase term. Afterward, Dongsheng et al. [35] defined CSVNS and its application in decision-making, which can handle phase terms but cannot mitigate their adverse effects. To fill up this gap, we presented the CSVNEPWA and CSVNEPWA, which enable us to make an appropriate selection as shown in Tables 9 and 10.

Table 9

Comparison of different approaches and operators

	SCORE VALUE BY USING SVNHFWA	SCORE VALUE BY USING SVNHFWG	SCORE VALUE BY USING SVNHFDWA	SCORE VALUE BY USING SVNHFDWG
p ₁	0.47454	0.267442	0.569793	0.169583
p ₂	0.523122	0.347964	0.596347	0.25366
p ₃	0.520915	0.326063	0.618722	0.232736
p ₄	0.544695	0.367541	0.606667	0.264385
p ₅	0.448348	0.290701	0.510086	0.206286

Table 10

Comparison of different approaches and operators

OPERATOR	RELATIONSHIP OF TWO ATTRIBUTIVE VALUES	RELATIONSHIP OF THREE ATTRIBUTIVE VALUES	RELATIONSHIP OF MULTIPLE ATTRIBUTIVE VALUES	BAD EFFECTS CAN BE REDUCED	THE PARAMETRIC METHOD MAKES METHOD FLEXIBLE	PHASE TERM
ZHANG & YE [35]	×	×	×	×	×	×
SERRANO- GUERRERO ET AL. [32]	×	×	×	×	×	×
ZHANG & YE [36]	✓	✓	✓	✓	✓	×
RAIZ ET AL. [29]	×	×	×	✓	×	×
DONGSHENG & TENGWU [10]	✓	✓	✓	×	✓	✓
CSVNEPWA (PROPOSED METHOD)	✓	✓	✓	✓	✓	✓
CSVNEPWG (PROPOSED METHOD)	✓	✓	✓	✓	✓	✓

The pictorial representation of score values for each alternative using AOs is shown in Figure 2.

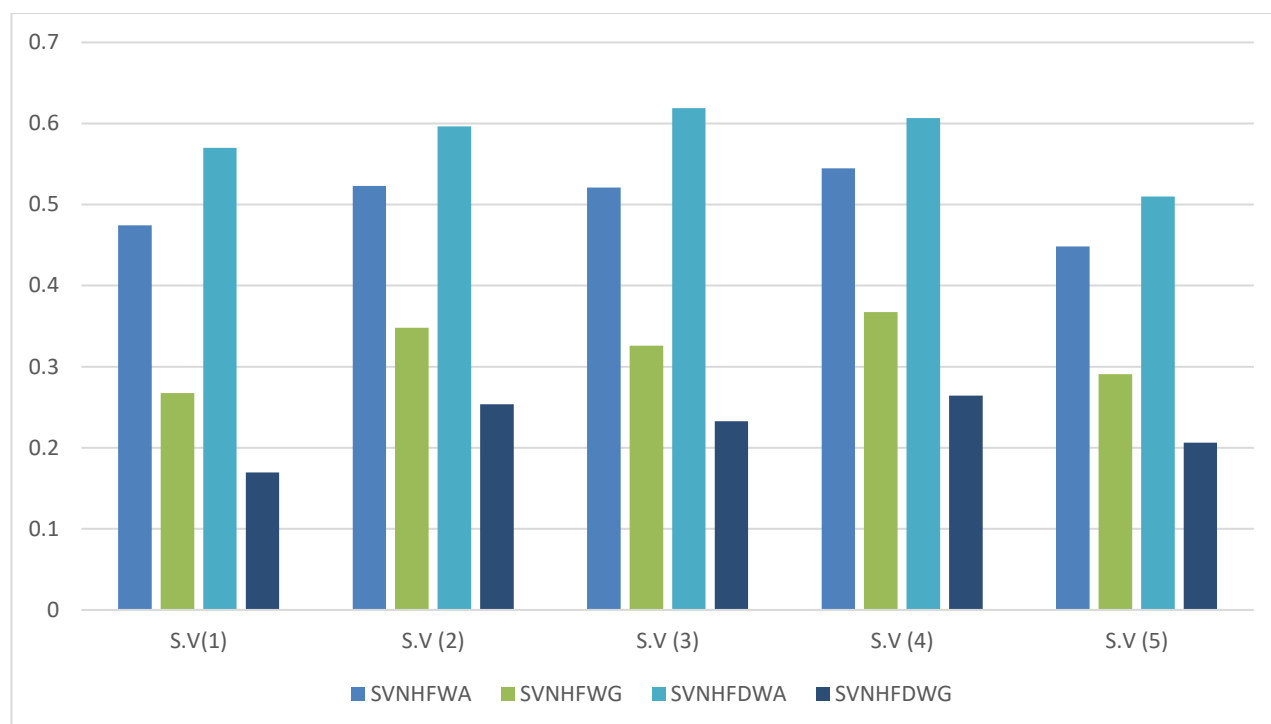


Fig. 2. Comparison of different approaches and operators

4.4. Advantages and Limitations of the proposed work

EPAO is a mathematical operator used in decision-making to aggregate individual preferences, opinions, or criteria based on their priority or importance. It is useful in situations where different criteria have varying levels of importance, and it is crucial to prioritize certain criteria over others to make precise decisions.

1. The main advantage of the proposed method is its ability to assign appropriate weight to the critical criteria, which leads to more useful and favorable outcomes.
2. The utilization of EPAO allows us flexible representation by adjusting the priority levels of various criteria and making it a versatile tool.
3. The EPAO has proven to be a remarkable operator in handling CSVN information within fuzzy set theory. However, the limitations of CSVN arise when experts provide information in the form of hesitant fuzzy, dual hesitant fuzzy, interval-valued fuzzy, etc., which renders the proposed method inadequate.

5. Conclusions

The decision-making strategy is a way of expressing challenges and incidents in a mathematical framework, and several theories have examined complicated and unreliable data by using a variety of methodologies and instruments. However, to handle this type of information, we have introduced CSVNEPAO, which aggregates complex data into a single value, enabling us to select the best alternative. In this paper, CSVNEPAO has been introduced to handle complex, unreliable data and support decision-making processes. In this study, we have successfully implemented the efficient and renowned Einstein AO by elaborating on the EPAO structure within the CSVN framework. The study explores the EPAO's characteristics, properties, and theorems, highlighting its unique features and

findings. An algorithm for addressing the MCGDM issue using the proposed operator was developed, demonstrating the efficacy of our approach and facilitating optimal selection among alternatives. To demonstrate the practical application of our proposed operator in resolving the MCGDM challenge, an illustrative example was presented. Moreover, a comparative analysis with preceding methodologies was conducted, showcasing the proposed approach's significance and superiority.

5.1. Contribution

The key contribution of this paper includes:

1. Demonstration of a decision-making algorithm specifically designed for the CSVN environment. CSVN offers a more comprehensive framework than IFS, capable of addressing truthiness, indeterminacy, and falsity simultaneously.
2. Proposed the EPAO, which includes operators like CSVNEPWA and CSVNEPWG, specifically tailored for CSVN elements. These operators effectively aggregate criteria into a single value, thereby enhancing decision-makers' data evaluation efficiency. Furthermore, a demonstration of the MCGDM algorithm based on the proposed AO enables efficient problem-solving in CSVN environments.
3. To illustrate the effectiveness of our method through a real-world example, demonstrating its practical applicability.
4. Lastly, a comparative analysis with existing methods was conducted, which highlighted the innovative strengths of our approach. Our method outperforms others in handling uncertainty, providing decision-makers with a valuable tool for navigating the complexities of everyday decision-making.

The evaluation confirms that shelter design significantly differentiates service quality outcomes on Istanbul's Metrobus system. The fully enclosed lounge emerged as the superior alternative, excelling in weather protection, safety, and comfort, while the smart digital shelter provided the best information services. These findings underscore the need for İETT to move beyond basic canopy structures toward multi-functional shelters that address both physical protection and digital passenger needs.

5.2. Limitations and potential challenges

The EPAO has proven to be a remarkable operator in handling CSVN information within fuzzy set theory. However, the limitations of CSVN arise when experts provide information in the form of hesitant fuzzy, dual hesitant fuzzy, interval-valued fuzzy, etc., which renders the proposed method inadequate. Moreover, the quality of data across all criteria is essential for effective evaluation. Hence, we propose the theory of complex single-valued neutrosophic hesitant fuzzy sets, complex single-valued neutrosophic dual hesitant fuzzy sets, complex interval-valued neutrosophic hesitant fuzzy sets, complex interval-valued neutrosophic dual hesitant fuzzy sets, etc., to address these gaps. Moreover, citizens were not involved in the evaluation process.

5.3. Future direction

In the future, we aim to extend this research by exploring the application of additional aggregation operators such as the BM operator, t-spherical, Frank AO, Dombi AO, and Aczel-Alsina aggregate operator across various domains and discuss its application in data analysis, supply chain

network, healthcare waste management, sustainable urban transportation, as this expansion will help us to understand their impact on everyday challenges more comprehensively, also recent artificial intelligence tools can be adopted to enhance the evaluation process [18, 27]. Moreover, citizens would be involved in the evaluation process.

Author Contributions

All authors contributed equally to preparing the manuscript, and they have read and agreed to the published version.

Availability of Data and Materials

All the data used and analyzed are available in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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